

THE DETECTION OF MARKOVIAN SEQUENCES OF SIGNALS

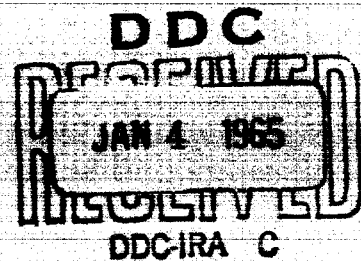
Morton P. Friedman and Edward C. Carterette

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The Detection of Markovian Sequences of Signals

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ABSTRACT

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The influence of constrained stimulus sequences on detection was studied in a two-alternative temporal forced-choice task with feedback. Three observers listened to a weak pure tone embedded in noise whose probabilities of occurrence and repetition in an interval were governed by a first-order Markov process. Each observer listened to examples of each of nine different Markov chains. Results were: (a) A single function relating detections to false alarms fitted individual sets of data well, in agreement with the theory of signal detectability, except (b) that detection was higher for more extreme repetition probabilities; (c) Responses depended strongly on the previous stimulus with (d) the dependence being peculiar to a given chain; (e) Detection probabilities increased during runs of signals in the same interval, yet (f) probability of detection on the first trial of a run in a given interval did not depend on the length of the preceding run in the other interval.

Author →

The Detection of Markovian Sequences of Signals

It is well known that the judgments of practiced observers in psychophysical experiments depend in an important way on both the previous sequence of stimuli and the observer's response to these stimuli. As long ago as 1860, Fechner¹ pointed out that there were substantial sequential response effects in the data of psychophysical experiments. For the most part, however, researchers interested in psychophysical problems have not concerned themselves with sequential effects.

Recent years have seen a rebirth of interest in the analysis of sequential effects in psychophysical judgments. This interest stems from two sources: first, the development of analytic techniques such as information theory and finite state methods; second, the emergence of theoretical interpretations of psychophysical experiments for which sequential effects are regarded as important aspects of the data.

The use of information measures in the analysis of sequential dependencies is illustrated by McGill's² experiment on serial effects in auditory judgments. On each of series of trials, one of four tones of equal intensity but differing in frequency was presented against a background of noise. The observer's task was to identify the tone presented on each trial. The difficulty of the judgment was varied by varying the noise level. Using uncertainty measures, McGill analyzed his data to determine the joint effect of previous responses (called presponse effects) and the stimulus in determining

judgments. McGill found a marked presponse effect for S/N ratios lower than 22 db. This corresponded to judgments at or below the threshold. As the S/N ratio increased, the presponse influence decreased. The nature of the presponse effect was the tendency of observers to avoid repeating responses on successive trials. In one series of trials run by McGill, no tones were ever presented. This can be regarded as a $-\infty$ S/N ratio. McGill argued that there was a continuum of presponse effects, running from the $-\infty$ S/N ratio condition in which observers simply guessed on each trial (although they were unaware that no tones were presented) to high S/N ratios in which the judgment was determined solely by the present stimulus. In a signal detection task, Speeth and Mathews³ obtained results on presponse effects which were essentially comparable to McGill's. Speeth and Mathews applied an interesting finite state approach to their data which promises to be quite useful. An excellent discussion of research on sequential response effects using uncertainty measures is contained in a recent book by Garner.⁴

The second recent source of interest in sequential effects has been the development of new theories of detection such as those of Atkinson,⁵ LaBerge,⁶ Luce,⁷ and Restle,⁸ some of which regard sequential response statistics as important aspects of detection responding. Like the theory of signal detectability,⁹ these new theories are judgmental theories in the sense that they deal explicitly with psychophysical and situational variables. But whereas the main

emphasis in theory of signal detectability has been on psychophysical variables, the major concern of these competing theories has been the influences of situational variables such as feedback and signal probability on detection responding.

These newer theories may be considered as an outgrowth of research on discrimination and probability learning in mathematical learning theory. In the probability learning or guessing experiment (reviewed by Estes¹⁰ and Anderson¹¹) the observer's task is simply to predict on each of series of trials which one of a set of events will occur. Following his response, he is told which event actually occurred on that trial. In the simplest sort of probability learning experiment, termed the "non-contingent" case, the sequence of events which occur on each trial is programmed independently of the observer's responses according to some probabilistic schedule. The non-contingent probability learning experiment is strikingly like the $-\infty$ S/N ratio condition (in which no tones were presented) in the McGill² experiment discussed earlier. The probability learning experiment may be said to represent an important limiting case of judgment, in which responding is completely determined by situational variables. Sequential statistics have played a major role in both the descriptive and theoretical analysis of the probability learning experiment. Assuming a continuity between the probability learning experiment and the usual psychophysical experiment in which the judgment is jointly determined by situational and psychophysical variables, it might be assumed that the analysis of sequential

statistics will prove of equal importance in understanding psychophysical experiments. The research in this laboratory on sequential effects in judgment is motivated by these considerations.

In the present experiment, we focused attention on the role of stimulus sequence in determining the sequence of judgments. To do this, we employed Markovian schedules of signal presentation, and compared the effects on detection performance of a number of different first-order Markov chain generators of trial sequences in a forced choice task.

METHOD

A conventional two-alternative temporal-forced-choice detection task was employed. On each trial, a gated 1000 c/s sinusoid appeared in one of two temporal observation intervals against a continuous background of wide-band Gaussian noise. The observer's task was to indicate which interval he thought contained the signal.

The temporal intervals were indicated to the observer by a display of pilot lamps. Each trial began with a one second warning interval followed by the two .75 second observation intervals. The observer made his response by pressing one of two pushbutton switches. As soon as the observer responded, one of two feedback lamps came on for one second to indicate the interval which contained the signal on that trial. After a 1.5 second intertrial interval, the next trial began.

On each trial, an electronic switch gated the signal on for 100 ms. (whose rise and fall times were equal to 25 ms.) in the middle

TABLE I

Transition Matrices for the Markov chains used to generate stimulus sequences and examples of sequences.

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Acoustical Society of America, New York City,
May 8, 1964

Chain A		$P(S_1) = 0.5$		Chain E		$P(S_1) = .5$	
Trial n+1		Trial n		Trial n+1		Trial n	
S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
$\begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix}$		$\begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix}$		$\begin{pmatrix} .2 & .8 \\ .8 & .2 \end{pmatrix}$		$\begin{pmatrix} .2 & .8 \\ .8 & .2 \end{pmatrix}$	
2122212221212121211211		2122212221212121211211		121212121122212121212121		121212121122212121212121	
22221111221121222211221		22221111221121222211221		2121222112212121212121		2121222112212121212121	
12212121222122211121111		12212121222122211121111		2122212121212121212122		2122212121212121212122	
222121122122222211112212		222121122122222211112212		2122212121212121212121		2122212121212121212121	
11222211121122121211121		11222211121122121211121		2122212121212121212121		2122212121212121212121	
21222222121222212111122		21222222121222212111122		21212111221121111111111		21212111221121111111111	
2111221212		2111221212		121212121121		121212121121	
Chain B		$P(S_1) = .8$		Chain F		$P(S_1) = .71$	
Trial n+1		Trial n		Trial n+1		Trial n	
S_1	S_2	S_1	S_2	S_1	S_2	S_1	S_2
$\begin{pmatrix} .8 & .2 \\ .8 & .2 \end{pmatrix}$		$\begin{pmatrix} .8 & .2 \\ .8 & .2 \end{pmatrix}$		$\begin{pmatrix} .8 & .2 \\ .5 & .5 \end{pmatrix}$		$\begin{pmatrix} .8 & .2 \\ .5 & .5 \end{pmatrix}$	
1122112111121111211111		1122112111121111211111		122122121112211121122		122122121112211121122	
11111111122121111111		11111111122121111111		1111221111111111111121		1111221111111111111121	
1111222121211111211221		1111222121211111211221		11211121112211112122		11211121112211112122	
11211111112111111121		11211111112111111121		21112211111211212211		21112211111211212211	
21112211111211212211		21112211111211212211		12111121112111111111		12111121112111111111	
121111111112221112		121111111112221112		211111111		211111111	
1121111211		1121111211		Chain G		$P(S_1) = .38$	
Chain C		$P(S_1) = .2$		Trial n+1		Trial n	
Trial n+1		Trial n		S_1	S_2	S_1	S_2
$\begin{pmatrix} .2 & .8 \\ .2 & .8 \end{pmatrix}$		$\begin{pmatrix} .2 & .8 \\ .2 & .8 \end{pmatrix}$		$\begin{pmatrix} .2 & .8 \\ .5 & .5 \end{pmatrix}$		$\begin{pmatrix} .2 & .8 \\ .5 & .5 \end{pmatrix}$	
12222222222221222121222		12222222222221222121222		221221222221221122121212		221221222221221122121212	
22212212222222222221212		22212212222222222221212		1222211121222221222121212		1222211121222221222121212	
222222122222212122222121		222222122222212122222121		21222122212121212212122		21222122212121212212122	
112221222222222222212222		112221222222222222212222		121222212121212121212121		121222212121212121212121	
122222222222222222212122		122222222222222222212122		22122122222222212211212121		22122122222222212211212121	
122121212122212222212221		122121212122212222212221		2112122212		2112122212	
2222112222		2222112222		Chain H		$P(S_1) = .62$	
Chain D		$P(S_1) = .5$		Trial n+1		Trial n	
Trial n+1		Trial n		S_1	S_2	S_1	S_2
$\begin{pmatrix} .8 & .2 \\ .2 & .8 \end{pmatrix}$		$\begin{pmatrix} .8 & .2 \\ .2 & .8 \end{pmatrix}$		$\begin{pmatrix} .5 & .5 \\ .8 & .2 \end{pmatrix}$		$\begin{pmatrix} .5 & .5 \\ .8 & .2 \end{pmatrix}$	
122222122122221111211111		122222122122221111211111		1111112121212121111121111		1111112121212121111121111	
11111222222222211122222		11111222222222211122222		121221212111211211211221		121221212111211211211221	
22222222222222222121112		22222222222222222121112		21121221121212121121222		21121221121212121121222	
22221112222222211111111		22221112222222211111111		111221212121211211212122		111221212121211211212122	
112111111222112221122212		112111111222112221122212		111212121		111212121	
2222222221		2222222221		Chain I		$P(S_1) = .29$	
Chain I		$P(S_1) = .29$		Trial n+1		Trial n	
Trial n+1		Trial n		S_1	S_2	S_1	S_2
$\begin{pmatrix} .5 & .5 \\ .2 & .8 \end{pmatrix}$		$\begin{pmatrix} .5 & .5 \\ .2 & .8 \end{pmatrix}$		$\begin{pmatrix} .5 & .5 \\ .2 & .8 \end{pmatrix}$		$\begin{pmatrix} .5 & .5 \\ .2 & .8 \end{pmatrix}$	
1212121211111112221121222		1212121211111112221121222		222122222222222221122221		222122222222222221122221	
222122222222222221122221		222122222222222221122221		122222222222222221122222		122222222222222221122222	
21122222122222211122222		21122222122222211122222		21122222122222211122222		21122222122222211122222	
22112212222211222211222		22112212222211222211222		2112222222222222211222		2112222222222222211222	
111222222222222221122221		111222222222222221122221		111222222222222221122221		111222222222222221122221	

of one of the observation intervals. The noise was not filtered. Observers listened binaurally over PDR-8 earphones in an anechoic chamber. The same signal and noise levels were used for all three observers. The calculated E/N_0 value was about 5.2.

Nine different first-order Markov chains were used to generate the sequence of intervals in which the signal appeared on successive trials. The chains differed in both the overall (apriori) probabilities of occurrence of the signal in the first interval, and in the probabilities of repetition of signal in a given interval on a pair of successive trials. Transition matrices for the nine Markov chains and examples of the sequences generated with them are shown in Table 1. A cell entry shows the probability that the signal

Table I follows

on the present [or $(n+1)$ st] trial is in interval 1 or 2 — the columns — given that the signal was in interval 1 or 2 on the preceding (or n th) trial — the rows. As an example consider Chain B. The first entry in the first row is .8, the second .2, which means that if the signal was in interval 1 on the last trial, it will be in interval 1 on this trial with probability .8, and in interval 2 on this trial with probability .2. The column entries for the second row are identical with those for the first row. This means that the transitions do not depend upon trials, that is to say, successive trials are independent. Furthermore, the overall probability of a signal in the first interval, indicated by $P(S_1)$, is equal to .8. Chains A, B, and C were all independent trial processes with $P(S_1)$ equal to .5, .8, and .2, respectively.

Chain D generated sequences with $P(S_1) = .5$ but with the probability of a repetition of the signal in the same interval on successive trials being .8. The sample sequences from this generator have longer homogeneous runs of signals in a given interval, compared with those from generator A, even though both A and D have $P(S_1)$ equal to .5. Chain E sequences had $P(S_1)$ of .5 but the probability of the signal interval alternating on successive trials was equal to .8. Thus, the sample sequence for E shows shorter homogeneous runs of signals than A or D with the same apriori probabilities. In Chain F, if the signal occurred in the first interval in trial n, it repeated in that interval only with probability .5. Chain H had the same structure as F with intervals reversed. In Chains G and I, one signal tended to repeat in an interval with probability only .2, while the other repeated with probability .5.

Three paid observers were used in the study, and all had at least 10,000 trials of practice on the various schedules before the actual data were taken. Each daily experimental session consisted of four 164-trial blocks. There was a five minute rest period between blocks in which observers left the experimental chamber. The first four trials of each block were "memory" trials, in which the signal level was increased 10 db. and the signal was presented twice in each observation interval in an 1221 order. Only the last 150 trials of each block were used in the analysis of the data. In a given daily session, only sequences generated by one of the nine chains were used. The nine chains were run through in random

order in successive experimental sessions, then were replicated in a new random order. For each of the nine chains, eight 160-trial sequences were constructed, and they were presented to each observer in a different random order. Additional sequences were constructed for use in the preliminary practice sessions. The sequences were generated by a computer program, and only those sequences were used which a χ^2 first-order Markov chain goodness-of-fit test could not reject at the .05 level.

The practice sessions which preceded the experiment also consisted of four 164-trial blocks, but sequences from chains A, B, C, D, and E were presented in a haphazard order in each session.

Observers were not told anything about the purpose of the experiment, were not informed about the various Markov chains used to generate the trial sequences, and were never told when conditions were shifted.

RESULTS AND DISCUSSION

Before pooling the data over blocks of trials, χ^2 tests for homogeneity of proportions were performed for each observer on the proportion of correct responses in each interval for the 8 blocks of trials for each chain. In only 3 of the 54 tests could the hypothesis of homogeneity be rejected at the .05 level, and there was no apparent pattern to the magnitude of the observed χ^2 's. It appears that under the conditions of this experiment, using feedback on each trial and shifting schedules during the preliminary sessions, observers come to adjust rather quickly to the context of a given type of sequence of trials. In all analyses, the data are pooled over blocks and

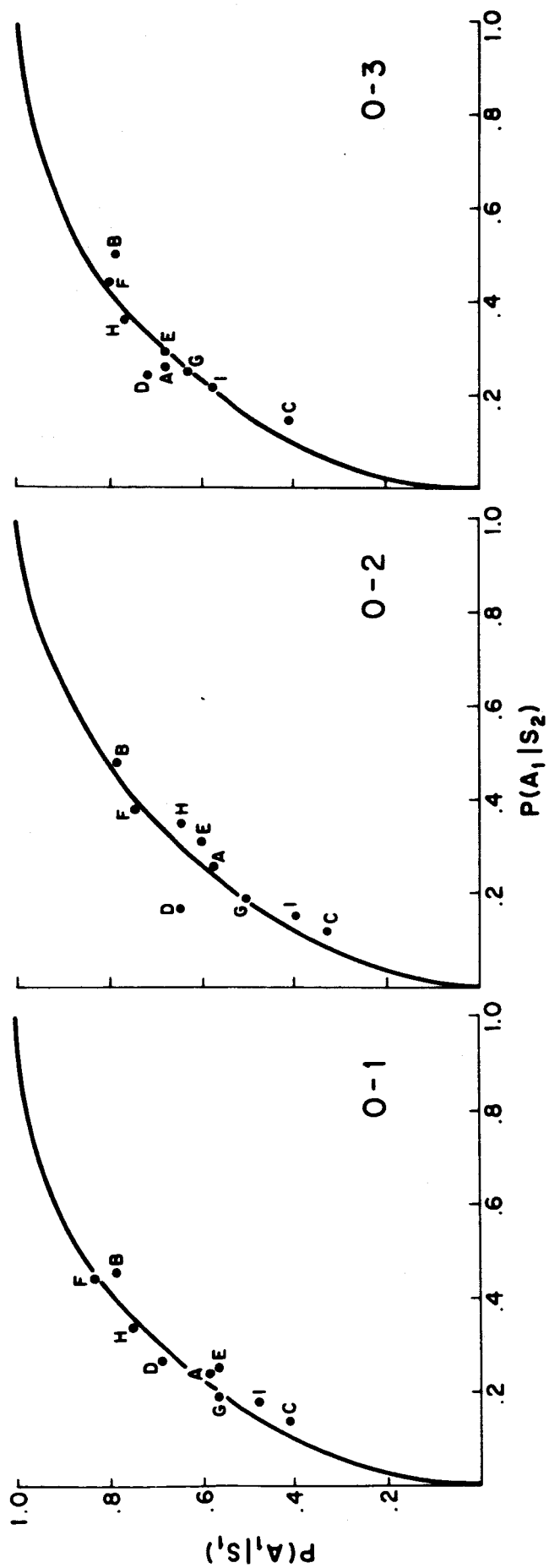
replications for each chain.

Mean detection rates on the various schedules for the three observers are shown on ROC plots in Figure 1. For illustrative

Figure 1 follows

purposes, we have fitted by eye functions derived from the theory of signal detectability to the data. There are a few things to note about these results.

First, although strong biases were induced by the various schedules, the single functions give a reasonable approximation to the data. Second, the pattern of biases induced by the various schedules are similar for the three observers. For the most part, the position of a given chain on the curve is determined by the overall apriori probabilities, irrespective of the stimulus dependencies. Thus, chains with high apriori probabilities of the signal being in the first interval are the highest points on the function, and chains with the lowest apriori probabilities of the signal in the first interval are the lowest points on the functions. The third point to be made about the data in Figure 1 is that there is some suggestion of greater sensitivity on schedule D, in which signals tend to repeat and long runs of signals occur in the same interval. Sequences from Chain D yield the largest estimate of $\underline{d'}$ for all observers. This apparent increase in sensitivity may simply be due to a decrease in the observer's uncertainty as to the onset time of the stimulus during a run of trials in which the signal recurred in the same interval. Egan, Greenberg, and Schulman¹² have shown signal uncertainty to

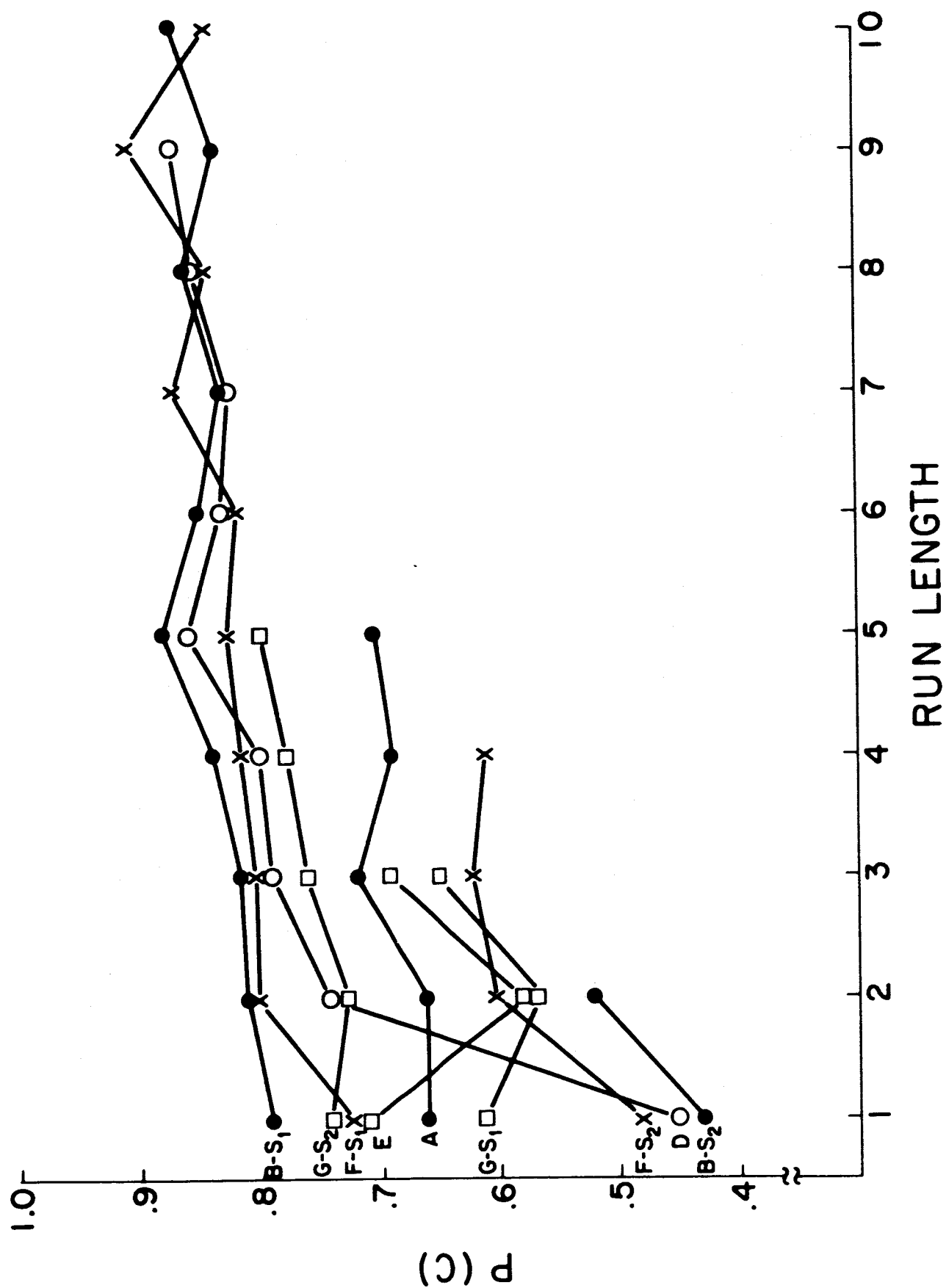


be a potent determiner of detection performance and Green⁹ has discussed theoretical implications of this for the theory of signal detectability. It is reasonable to assume that, in responding to sequences generated by Chain D which tended to have long runs of trials in which the signal recurred in the same interval, observers' uncertainty as to the onset time of the signal would decrease during a run and detection would be enhanced.

Of greater interest to the study of stimulus dependencies is responding during runs of trials on which the signal recurs in the

Figure 2 follows

same interval. Figure 2 shows the proportion of correct responses to the various chains during such runs. Since the data from the three observers were similar, we have averaged over the three observers. And, whenever possible, we have combined data within and between symmetrical chains, such as B and C, and G and I. Thus, we have combined frequencies of correct responding for both intervals in Chain A. Likewise, the points labeled B-S₁ are from the combined tabulations of runs of signals in the first interval for Chain B and runs of signals in the second interval for Chain C. Data are presented only for those points for which there are at least 125 observations. Because the number of possible observations of runs depends on the structure of the chain, the number of points for each chain differ. Points for runs of length k include observations for the kth trial of runs longer than k. The gist of Figure 2 seems to be as follows: For schedules in which successive trials are independent,



there is a slight increase in correct responding during a run. Thus for Chain A, with apriori probabilities of .5, and independent trials, the mean detection rate for the first trial of a run is .66, and increases to .70 on the fifth trial of a run of signals in the same interval. On Chain B, with an apriori probability of signal occurrence in the first interval equal to .8, the curve labeled B-S₁ shows the detection rate during runs of trials in which the signal recurred in the first interval. On the first trial of a run, the mean detection rate was .79, and on the 11th trial of the run, the mean detection rate was .84.

For chains in which there were dependencies in the stimulus sequences, detection responding mirrored these dependencies markedly. For example, consider Chain D, in which $P(S_1)$ was .5, but signals recurred in the same interval with probability .8 on successive trials. Figure 2 shows that on the first trial of a run, the detection rate was .45, but that it increased rapidly to about .85. Responding to signals in the first interval for Chain F, (the points labeled F-S₁) show the same effect to a lesser extent. For Chain E, where the signal tended to alternate in the two intervals on successive trials, the detection rate on the first trial of a run was .70, and dropped to .58 on the second trial of a run, again mirroring the dependencies in the signal sequence. The same effect occurs in responding to runs of signals in the first interval for Chain G (the points labeled G-S₁). These results make it clear that signal events on preceding trials are important determiners of trial-

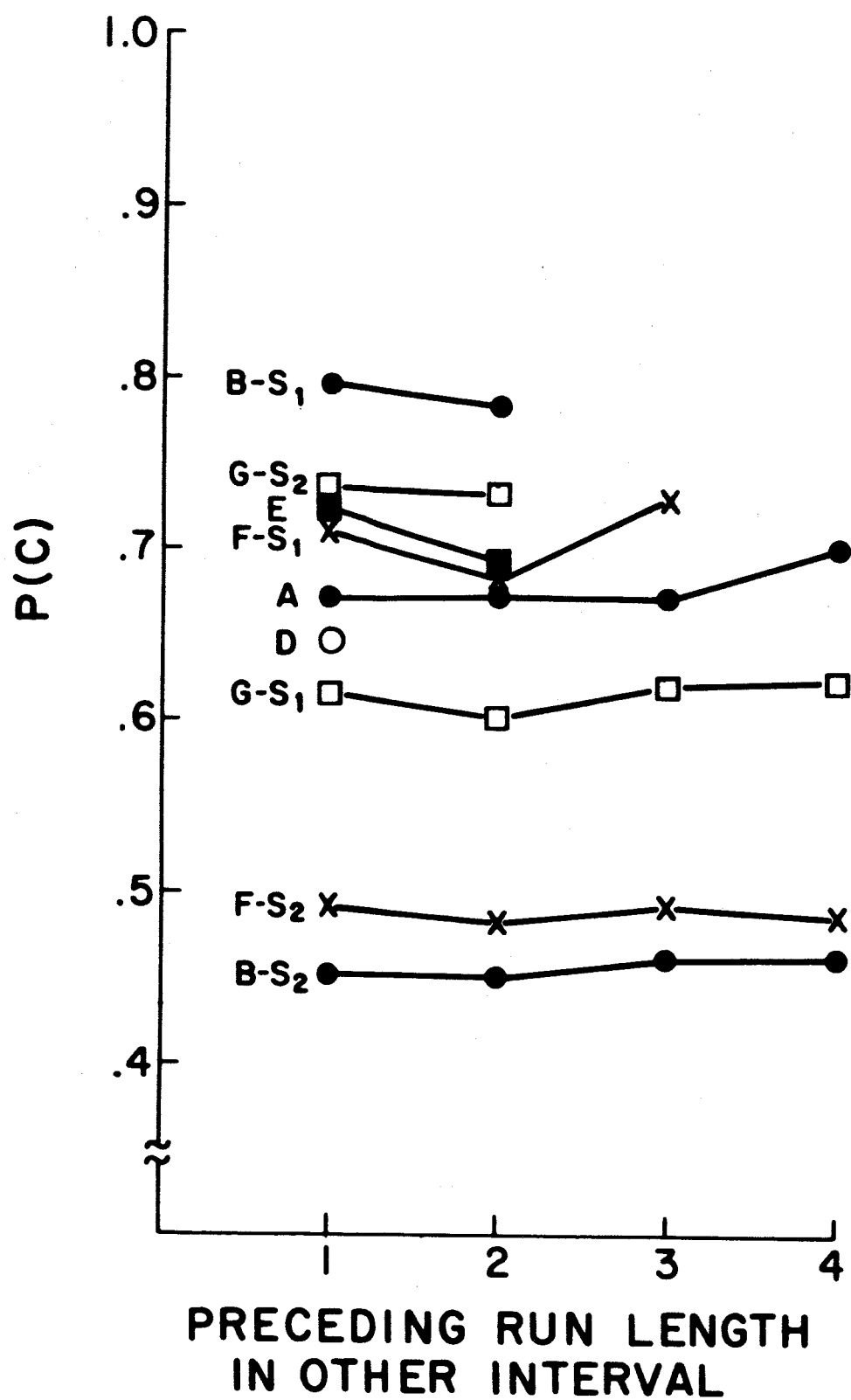
to-trial response biases.

Another statistic of interest in gauging the influence of stimulus dependencies on responding is the detection rate on those trials which break a run of signals in a given interval. Figure 3

Figure 3 follows

shows the detection rate on the first trial of a run of signals in a given interval as a function of the length of the preceding run of signals in the other interval. The points are labeled and the data were tabulated in the same way as the run data in Figure 2. The result here is quite clear: There is little or no effect of the length of the preceding run on detection responding, excepting a slight effect for chains which tend to alternate. These data indicate that when the signal switches intervals, the immediately preceding trial is the important determiner of bias.

We have discussed only the effects of stimulus contingencies on responding. The observed dependence of responses on preceding responses is quite small compared to the effects of preceding stimuli. This is probably because of our use of feedback on every trial. Research in this laboratory¹³ and elsewhere¹⁴ indicates that when feedback is employed, it is the important determiner of sequential responding; when there is no feedback, then previous responses appear to be important. The nature of the dependence when there is no feedback is a tendency to alternate responses, as in McGill's² study, or to repeat responses, as in a recent study by Parducci and Sandusky.¹⁴ Much clarification is needed of the variables



which control responding in the absence of feedback.

The theory of signal detectability has not yet been extended to deal with sequential response effects, but Atkinson's⁵ detection model has been developed in enough detail to allow exact quantitative predictions of sequential statistics. It is essentially a threshold theory of the sort proposed by Blackwell.¹⁵ Atkinson formulated his theory in the language of stimulus sampling theory, but we shall discuss it in more general terms here. As applied to the yes-no experiment with feedback, it is assumed that on every trial, the observer either correctly detects the presence or absence of the signal, or he is uncertain whether the signal was presented. If he is uncertain, he guesses. According to Atkinson's model, sequential effects are due to the fact that the guessing probabilities change in a trial-to-trial fashion as a function of the feedback. If a signal is presented on a given trial, then the probability of guessing yes associated with the uncertain state increases; if the feedback indicates that no signal was presented, then the probability of guessing yes decreases. Atkinson's theory has had success in accounting for sequential effects as well as mean response proportions in both feedback and non-feedback visual¹⁶ and auditory¹⁷ detection experiments.

However, Atkinson's theory is not consistent with our results. The main difficulty seems to be with predictions concerning responding during homogeneous runs of signals in the same interval. Atkinson's prediction is, that during homogeneous runs, detection

rate should continue to increase, and assuming that sensitivity is constant, responding on all schedules should approach the same asymptote. This is clearly not the case in Figure 2. Similarly, Atkinson's theory predicts that the probability of a correct detection on the first trial of a run of signals in an interval should be a decreasing function of the preceding run length in the other interval. The data presented in Figure 3 indicates that responding on the first trial of a run is independent of the length of the preceding run in the other interval.

We do not regard these negative results in themselves as particularly damaging to Atkinson's model. For one thing, it is clear that if Atkinson's theory about changes in guessing probability when in the uncertain state were modified to put greater weight on the outcome of the preceding trial, then predictions would be more in line with our results. Also, sequential analyses of data from other experiments in this laboratory^{13, 17} have yielded results which are at least in qualitative agreement with Atkinson's model. The main difference between this study and the others seems to be that in the present study, observers were exposed to a much wider variety of probabilistic schedules. Our impression is that under the conditions of this experiment, observers quickly adjusted to the context of a new schedule, and the observed sequential effects are determined by a few short term strategies that the observers are using. The quantification of a theory incorporating these notions is difficult, but the finite state methods used by Speeth and Mathews³ seem to offer a reasonable approach.

References

1. G. T. Fechner, Elemente der psychophysik, (Breitkopf and Hartel, Leipzig, 1860)
2. W. McGill, "Serial Effects in Auditory Threshold Judgments," J. exp. Psychol. 53: 297-303 (1957).
3. S. D. Speeth and M. V. Mathews, "Sequential Effects in the Signal-Detection Situation," J. Acoust. Soc. Amer. 33: 1046-1054 (1961).
4. W. R. Garner, Uncertainty and Structure as Psychological Concepts. (Wiley, New York, 1962)
5. R. C. Atkinson, "A Variable Sensitivity Theory of Signal Detection," Psychol. Rev. 70: 91-106 (1963)
6. D. LaBerge, "A Recruitment Theory of Simple Behavior," Psychometrika. 27: 375-396 (1962)
7. R. D. Luce, "A Threshold Theory for Simple Detection Experiments," Psychol. Rev. 70: 61-79 (1963)
8. F. Restle, Psychology of Judgment and Choice (Wiley, New York, 1961).
9. D. M. Green, "Psychoacoustics and Detection Theory," J. Acoust. Soc. Amer. 32: 1189-1203 (1960)
10. W. K. Estes, "Probability Learning" in Categories of Human Learning, edited by A. Melton, (Academic Press, New York, 1964.)
11. N. H. Anderson, "An Evaluation of Stimulus Sampling Theory" in Categories of Human Learning, edited by A. Melton, Academic Press, New York, 1964)
12. J. P. Egan, G. Z. Greenberg, and A. Schulman, "Interval of Time Uncertainty in Auditory Detection," J. Acoust. Soc. Amer. 33: 771-778 (1961).
13. E. C. Carterette and R. C. Atkinson, "Sequential Phenomena in Signal Detection," J. Acoust. Soc. Amer. 34: 172 A (1962).
14. A. Parducci and A. Sandusky, "Distribution and Sequence Effects in Judgment." J. exp. Psychol. in press (1964).

15. H. R. Blackwell, "Psychophysical Thresholds: Experimental Studies of Methods of Measurement. " Bull. Engng. Res. Inst. Ann Arbor, No. 36 (1953).
16. R. A. Kinchla "A Learning Factor in Visual Discrimination, " in Studies in Mathematical Psychology, edited by R. C. Atkinson (Stanford Univ. Press, Stanford, 1963).
17. R. C. Atkinson, E. C. Carterette and R. A. Kinchla, "Sequential Phenomena in Psychophysical Judgments: A Theoretical Analysis" IRE Transactions on Information Theory, IT-8, 5155-62 (1962).

Figure Captions

Figure 1. ROC plots for the three observers.

Proportion of correct responses in Interval 1 [$P(A_1 S_1)$], are plotted against the proportion of incorrect responses in Interval 2 [$P(A_1 S_2)$]. Points labeled with letters are from different Markov chain generators. The smooth curves are derived from the theory of signal detectability. The d' values for the three observers were 1.08 for 0-1 and 0-3, and .94 for 0-2.

Figure 2.

Proportion of correct responses [$P(C)$] averaged over the three observers as a function of run length for the various Markov chain generators.

Figure 3.

Proportion of correct responses [$P(C)$] on the first trial of a run as a function of the preceding run length in the other interval shown for the various Markov chain generators. Data are averaged over the three observers.